The Mirror Symmetry of Matter and Antimatter

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Abstract. Physical processes involving weak interactions have mirror images which can be mimicked in the natural universe only by exchanging matter and antimatter. This experimental observation is easily explained by the hypothesis that spatial inversion exchanges matter and antimatter. Yet according to conventional theory, the parity operator P does not exchange matter and antimatter but instead yields phenomena which have never been observed. We examine the conventional derivation of the Dirac parity operator and find that it incorrectly identifies the matrices associated with probability density (γ^0 rather than the identity matrix I) and current (γ^i rather than $\gamma^5\sigma_i$). This illusory functional dependence incorrectly requires that γ^0 preserve its sign under spatial inversion. This requirement results in a mixed-parity vector space defined relative to velocity, which is otherwise isomorphic to the spatial axes. We derive a new spatial inversion operator M (for mirroring) by introducing a pseudoscalar unit imaginary and requiring that for any set of orthogonal basis vectors, all three must have the same parity. The M operator is a symmetry of the Dirac equation. It exchanges positive and negative energy eigenfunctions, consistent with all experimental evidence of mirror symmetry between matter and antimatter. This result provides a simple reason for the apparent absence in nature of mirror-like phenomena, such as right-handed neutrinos, which do not exchange matter and antimatter. A new time reversal operator B (for backward) is also derived to be consistent with the geometry of the polar vectors inverted by the M operator.

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1. Introduction

It has long been the conventional assumption that spatial inversion (mirror imaging and 180 degree rotation about the reflected axis) does not change electrical charge. The conventional Dirac parity operator for fermions also does not exchange matter and antimatter. Yet experimental evidence invariably demonstrates that physical processes which are left-handed for matter are right-handed for antimatter, and vice versa. In this paper we will consider the possibility that matter and antimatter differ only by spatial inversion. To avoid confusion, we will keep the name P for the conventional Dirac parity operator which inverts only position, velocity, and one of two other vectors orthogonal to velocity. We will denote by M an operator which inverts position, velocity, and both additional vectors which, together with velocity, form an orthogonal basis in three dimensions.

With regard to symmetries, we consider an operation to be a symmetry of nature if it exchanges one physically realizable process with another (or with itself), even if the two processes do not occur with equal frequency in the universe. For example, an exchange of an isolated electron and positron would be regarded as a mass-conserving symmetry because both occur in nature, even though electrons are more common.

Historically, parity conservation was a fundamental assumption of physics. Any physical bias toward right- or left-handed processes would be completely arbitrary and therefore unjustifiable. However, Lee and Yang [1] proposed that weak interactions may violate parity conservation, and experiments by Wu et al. [2] demonstrated that beta decay exhibits left-right asymmetry. This asymmetry has been interpreted as implying parity violation, although Lee and Yang mention that their theory could be consistent with parity conservation if protons and electrons are not identical to their mirror images.

Wu's experiment involved beta decay of Cobalt 60. A beta electron emitted anti-parallel to nuclear spin would have a mirror particle emitted parallel to nuclear spin, since velocity is a polar vector whereas spin is an axial vector. Using the conventional parity operator for Dirac wave functions and assuming mirror symmetry, it was predicted that the mirror image process would be consistent with beta decay of Cobalt 60. This would only be possible if the beta electrons were emitted with equal likelihood parallel and anti-parallel to the nuclear spin. This was the prediction of conventional theory.

In fact, the beta electrons in decay of Cobalt 60 are all emitted anti-parallel to the nuclear spin. Hence the mirror image process is not consistent with beta decay of Cobalt 60. Rather, the mirror process proceeds exactly like beta decay of antimatter. Hence the conventional theory failed to predict the correct result of the experiment.

There are two possible explanations for this result: (1) spatial inversion does not exchange matter and antimatter, and beta decay violates mirror symmetry, or (2) spatial inversion exchanges matter and antimatter consistent with mirror symmetry, implying that the conventional Dirac parity operator is incorrect.

Standard theory chooses the first explanation and calls it "parity violation". This choice not only abandons what used to be universally regarded as a fundamental symmetry, it requires the existence of new particles which have never been observed (either 'mirror matter' itself or other particles invented to explain why such 'mirror matter' is never observed).

In this paper we will show that the second explanation not only provides a simple interpretation of the experimental evidence, it also invokes a spatial inversion operator which is more consistent with ordinary geometry than the conventional Dirac "parity" operator P.

Parity violation was first suspected in the decay of charged K mesons. The meson decay products of K^+ (either $\pi^+\pi^0$ or $2\pi^+\pi^-$) have opposite eigenvalues of the conventional parity operator. However, if spatial inversion were to exchange matter and anti-matter, then none of the initial or final states would be spatial inversion eigenfunctions, and there would be no violation of mirror symmetry. Decay of K^- appears experimentally to have perfect mirror symmetry with decay of K^+.

In the decay of neutral kaons, the value of the theoretical PC operator (parity and charge conjugation C) apparently changes sign (or is ambiguous). In its mesonic decays, the long-lived neutral kaon state (K_L) decays into either $3\pi^0$ (PC=-1) or $2\pi^0$ (PC=+1) or $\pi^+\pi^-$ (PC=+1). Exchanging matter and anti-matter does not change the distribution of particles in any of these final states. Thus PC violation is claimed in spite of the fact that experimentally the mirror symmetry between matter and anti-matter is evidently maintained. This situation again suggests that at least one of the operators has been defined incorrectly.

The so-called "direct" PC violation in neutral kaon decay involves comparison of decay rates of the long- and short-lived neutral kaons into $2\pi^0$ or $\pi^+\pi^-$. Again, the claim of PC violation is based on theoretical calculations, and not on any observed mirror asymmetry between matter and anti-matter.

In its semi-leptonic decay modes, K_L is slightly more likely to produce a positron rather than electron. This result does indicate unequal amounts of matter and anti-matter in the composition of K_L. Nonetheless, $\bar{K}_L$ would decay with an equally slight preference toward electrons and there is no evidence of any mirror asymmetry between matter and antimatter.

A straightforward interpretation of the experimental observations is that matter and antimatter are related by spatial inversion. However, other theoretical considerations have led physicists to believe that spatial inversion would not exchange matter and anti-matter, in spite of the experimental evidence. The purpose of this paper is use algebraic principles to derive a Dirac operator for spatial inversion which exchanges matter and antimatter, consistent with experiment. This operator has been reported previously [3], but the derivation given here is more thorough, particularly in the analysis of velocity-representation space and vectors orthogonal to velocity.

We first show that the derivation of the conventional Dirac parity operator relies on an incorrect identification of the Dirac matrices associated with the probability current 4-vector. Next, we identify three matrices which have the same algebra as the Pauli matrices. We call this space "velocity-representation space" because rotations in this space change the matrix representation of the velocity operator. Each matrix in this space is associated with a vector in real space. We will assume that all three of these vectors are polar vectors. The conventional parity operator includes a 180 degree rotation in this vector space rather than an inversion. We derive a new spatial inversion operator M which inverts position and all three matrices of the velocity-representation space. The new spatial inversion operator exchanges matter and anti-matter, and is therefore consistent with mirror symmetry in every known physical process. We also define a new time reversal operator B which reverses the spin and all three vectors of the velocity-representation space.

2. Conventional parity operator

Dirac's original equation for a free particle has the form:

$$\partial_t \psi + c\gamma^5 \sigma_i \partial_i \psi = -\mathrm{i}\Omega\gamma^0\psi \tag{1}$$

where $\Omega = mc^2/\hbar$. One representation for the matrices γ^μ is:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \gamma^i = \gamma^0\gamma^5\sigma_i \tag{2}$$

where σ^i are the spin matrices (with $\bar{\mathrm{i}}$ being the true scalar unit imaginary):

$$\sigma_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -\bar{\mathrm{i}} & 0 & 0 \\ \bar{\mathrm{i}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{\mathrm{i}} \\ 0 & 0 & \bar{\mathrm{i}} & 0 \end{pmatrix},$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{3}$$

Multiplying the Dirac equation by $\psi^\dagger$ and adding the Hermitian conjugate equation yields a continuity equation:

$$\partial_t \left[\psi^\dagger\psi\right] + c\nabla \cdot \left[\psi^\dagger\gamma^5\boldsymbol{\sigma}\psi\right] = 0 \tag{4}$$

This relationship is sufficient to establish the probability density $(\psi^\dagger\psi)$ and current $(\psi^\dagger\gamma^5\boldsymbol{\sigma}\psi)$ as the components of a Lorentz four-vector.

Although the above analysis is satisfactory, it is currently fashionable to multiply each term in the original Dirac equation (1) by γ^0 to obtain:

$$\gamma^0\partial_t\psi + c\gamma^0\gamma^5\sigma_i\partial_i\psi \equiv \gamma^\mu\partial_\mu\psi = -\mathrm{i}\Omega\psi \tag{5}$$

This procedure cannot have any effect on the transformation properties of the Dirac matrices or wave function.

The conventional parity operator P is assumed to have the form: $P\psi(\mathbf{r}) = U\psi(-\mathbf{r})$. It is derived from the requirement that the Dirac equation in the form (5) be covariant with respect to the transformation:

$$\gamma^0\partial_t\left[U\psi(-\mathbf{r})\right] + c\gamma^i\partial_i\left[U\psi(-\mathbf{r})\right] = -\mathrm{i}\Omega U\psi(-\mathbf{r}) \tag{6}$$

Inverting the parity operator yields:

$$U^{-1}\gamma^0 U\partial_t\psi(\mathbf{r}) - cU^{-1}\gamma^i U\partial_i\psi(\mathbf{r}) = -\mathrm{i}\Omega U^{-1}U\psi(\mathbf{r}) \tag{7}$$

Equivalence with the original Dirac equation requires:

$$U^{-1}\gamma^0 U = \gamma^0 \tag{8}$$

$$U^{-1}\gamma^i U = -\gamma^i \tag{9}$$

$$U^{-1}U = 1 \tag{10}$$

These conditions are satisfied by $U = \gamma^0$. Within an arbitrary phase factor the conventional parity operator is therefore:

$$P\psi(\mathbf{r}) = \gamma^0\psi(-\mathbf{r}) \tag{11}$$

There are two problems with this derivation. First, the form $P\psi(\mathbf{r}) = U\psi(-\mathbf{r})$ is not the most general possible operator. For example, the conventional charge conjugation operator includes complex conjugation. More general requirements for the parity operator would be:

$$P^{-1}\{P\psi\} = \psi \tag{12}$$

$$P^{-1}\left\{\gamma^5\sigma_i P\psi\right\} = -\gamma^5\sigma_i\psi \tag{13}$$

$$P^{-1}\left\{\mathrm{i}\gamma^0\Omega P\psi\right\} = \mathrm{i}\gamma^0\Omega\psi \tag{14}$$

With these constraints it is clear that γ^0 may be inverted if the sign of the unit imaginary (i) is also changed by spatial inversion. One should also consider the possibility that Ω is a pseudoscalar. If so then the parity operator would be $P\psi(\mathbf{r}) = \mathrm{i}\gamma^0\gamma^5\psi(-\mathbf{r})$.

The second problem is that the conventional parity operator treats the matrix γ^0 as a temporal component of a four-vector which must preserve its sign under spatial inversion. This illusion is maintained by rewriting the probability density and current components as $\bar{\psi}\gamma^0\psi$ and $\bar{\psi}\gamma^i\psi$, respectively, with $\bar{\psi} = \psi^\dagger\gamma^0$. This change of notation does not change the fact, however, that the probability density is independent of γ^0. The matrix associated with the temporal part of the probability current 4-vector is the identity matrix, not γ^0. This is a serious conceptual flaw in the conventional derivation of the parity operator.

Since the 4-vector $(\psi^\dagger\psi,\ \psi^\dagger\gamma^5\boldsymbol{\sigma}\psi)$ is indeed Lorentz-covariant, there is absolutely no basis for the claim that γ^0 is a temporal component. On the contrary, we will show that γ^0 is geometrically related to wave velocity and should be inverted by spatial inversion. The resulting spatial inversion operator inverts all of the terms in the modified Dirac equation (5).

3. New spatial inversion operator

In discussing spatial inversion, it will be necessary to define two different unit imaginary numbers. As defined above, the product of spin matrices is a true scalar with respect to spatial inversion:

$$\bar{\mathrm{i}} = \sigma_1\sigma_2\sigma_3 \tag{15}$$

The σ-matrices are not involved in spatial inversion, which inverts the wave velocity but not the spin. We can identify three other matrices associated with wave velocity which have the same algebra as the σ-matrices but are inverted by spatial inversion.

We start by defining a pseudoscalar imaginary $\tilde{\mathrm{i}}$. Its role in the Dirac equation is as follows:

Consider the three matrices $(\gamma^5, \gamma^4, \gamma^0)$ with $\gamma^4 \equiv \tilde{\mathrm{i}}\gamma^5\gamma^0$ (or equivalently, $\tilde{\mathrm{i}} \equiv \gamma^5\gamma^4\gamma^0$). These matrices have the same commutation relations as the Pauli matrices, and therefore may be interpreted as forming an oriented vector space. These matrices may be represented in terms of the 2×2 identity matrix I_2:

$$\gamma^5 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \quad \gamma^4 = \begin{pmatrix} 0 & -\tilde{\mathrm{i}}I_2 \\ \tilde{\mathrm{i}}I_2 & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} \tag{16}$$

Each of these matrices is a reflection operator for the others:

$$\gamma^\alpha\gamma^\beta\gamma^\alpha = (2\delta_{\alpha\beta} - 1)\,\gamma^\beta \tag{17}$$

Passive rotations in this vector space constitute changes of velocity 'representation'. These matrices define directions relative to the velocity vector $\langle c\gamma^5\boldsymbol{\sigma}\rangle \equiv c\psi^\dagger\gamma^5\boldsymbol{\sigma}\psi/|\psi^\dagger\psi|$. One can also define absolute vectors $(\langle c\gamma^5\boldsymbol{\sigma}\rangle, \langle c\gamma^4\boldsymbol{\sigma}\rangle, \langle c\gamma^0\boldsymbol{\sigma}\rangle)$ [4]. If the wave function is an eigenfunction of velocity aligned with a spatial axis x_{v} so that $\gamma^5\sigma_{\mathrm{v}}\psi = \psi$, then (using $\sigma_{\mathrm{v}}^2 = 1$):

$$\psi^\dagger c\gamma^0\sigma_{\mathrm{v}}\psi = \left[\psi^\dagger\gamma^5\sigma_{\mathrm{v}}\right] c\gamma^0\sigma_{\mathrm{v}} \left[\gamma^5\sigma_{\mathrm{v}}\psi\right] = -\psi^\dagger c\gamma^0\sigma_{\mathrm{v}}\psi = 0 \tag{18}$$

$$\psi^\dagger c\gamma^4\sigma_{\mathrm{v}}\psi = \left[\psi^\dagger\gamma^5\sigma_{\mathrm{v}}\right] c\gamma^4\sigma_{\mathrm{v}} \left[\gamma^5\sigma_{\mathrm{v}}\psi\right] = -\psi^\dagger c\gamma^4\sigma_{\mathrm{v}}\psi = 0 \tag{19}$$

These results follow from the fact that γ^5 is a reflection operator for both γ^0 and γ^4, and the only number equal to its negative is zero. Therefore $\langle\gamma^0\boldsymbol{\sigma}\rangle$ and $\langle\gamma^4\boldsymbol{\sigma}\rangle$ are indeed perpendicular to velocity $\langle\gamma^5\boldsymbol{\sigma}\rangle$ for velocity eigenfunctions. For example, in our notation the wave function $(1\quad 0\quad 0\quad 1)^{\mathrm{T}}$ is a simultaneous eigenfunction of $\gamma^5\sigma_1$, $\gamma^4\sigma_2$, and $\gamma^0\sigma_3$. Therefore the three vectors $\langle\gamma^5\boldsymbol{\sigma}\rangle$, $\langle\gamma^4\boldsymbol{\sigma}\rangle$, and $\langle\gamma^0\boldsymbol{\sigma}\rangle$ are mutually orthogonal vectors in three dimensional space, at least for velocity eigenfunctions. The vector $\left\langle \gamma^5 \boldsymbol{\sigma} \right\rangle$ is parallel to $\hat{\mathbf{x}}_1$. Rotation of the vector $\left\langle \gamma^5 \boldsymbol{\sigma} \right\rangle$ by -90 degrees about the relative vector γ^4 yields $\left\langle \gamma^0 \boldsymbol{\sigma} \right\rangle$, which is parallel to $\hat{\mathbf{x}}_3$. This is of course the same as rotation of $\hat{\mathbf{x}}_1$ by -90 degrees about $\hat{\mathbf{x}}_2$, which is associated with the matrix σ_2. It is clear that for velocity eigenfunctions, the relative vectors represented by $(\gamma^5, \gamma^4, \gamma^0)$ form a vector space equivalent with respect to rotations to the absolute vectors represented by $(\sigma_1, \sigma_2, \sigma_3)$. We therefore assume that all three vectors $\left\langle \gamma^5 \boldsymbol{\sigma} \right\rangle$, $\left\langle \gamma^4 \boldsymbol{\sigma} \right\rangle$, and $\left\langle \gamma^0 \boldsymbol{\sigma} \right\rangle$ are polar vectors so that the vector space $(\gamma^5, \gamma^4, \gamma^0)$ does not have mixed parity.

The matrix factor γ^0 in the conventional parity operator represents a rotation by 180 degrees about the γ^0 axis ($\hat{\mathbf{x}}_3$ in our example). This operation inverts only two of the three orthogonal vectors associated with velocity.

Compare this situation with classical transverse waves in a solid. We could define an operator (analogous to the Dirac P operator) which reflects the equilibrium position of each point in the solid, and also reflects the wave velocity direction. We also invert local displacements and velocities along one of the two axes perpendicular to the wave velocity. The resulting "reflected" wave would propagate along just as one would expect for a spatially inverted wave. But of course the operator we defined is not the spatial inversion operator, because we failed to invert one of the axes of the local displacement and velocity of the solid medium (in total we inverted two of the three local axes, corresponding to a 180 degree rotation about the third axis). Similarly, the Dirac P operator inverts the "wave" (or "particle") velocity direction, but inverts only one of two other quantities which are geometrically related to the "wave" velocity (a 180 degree rotation in the velocity-representation space). We will derive a new spatial inversion operator which inverts all three vectors ($\left\langle \gamma^5 \boldsymbol{\sigma} \right\rangle$, $\left\langle \gamma^4 \boldsymbol{\sigma} \right\rangle$, and $\left\langle \gamma^0 \boldsymbol{\sigma} \right\rangle$) associated with velocity.

The spin matrices σ_i are components of a pseudovector and should not be inverted. Spatial inversion must be accomplished by inverting the three relative matrices $(\gamma^5, \gamma^4, \gamma^0)$. The unit imaginary $\tilde{\mathrm{i}}$ associated with these matrices must be a pseudoscalar, as assumed above. The unit imaginary associated with mass is assumed to be a pseudoscalar since it is multiplied by γ^0 in the original Dirac equation. The roles of the different imaginaries can be clarified by factoring the Dirac wave function in a manner similar to that of Hestenes [5]:

$$\psi\left(\mathbf{r}\right) = a^{1/2} \exp\left(\bar{\mathrm{i}}\sigma_i \phi_i\right) \exp\left(\gamma^5 \sigma_j \alpha_j\right) \exp\left(\tilde{\mathrm{i}}\gamma^5 \zeta\right) \psi_0 \tag{20}$$

where a is an amplitude, ϕ_i are orientation angles, α_j are velocity boost parameters, and ψ_0 is a constant column vector. The angle ζ determines the orientation of the vectors $\left\langle \gamma^4 \boldsymbol{\sigma} \right\rangle$, and $\left\langle \gamma^0 \boldsymbol{\sigma} \right\rangle$.

Next we define a new wave function in which all imaginary pseudoscalar factors are inverted: $\psi^{\#}\left(\tilde{\mathrm{i}}\right) = \psi\left(-\tilde{\mathrm{i}}\right)$. This pseudoscalar conjugation operation differs from complex conjugation, which inverts both scalar and pseudoscalar imaginaries. Pseudoscalar conjugation inverts $\langle\gamma^4\rangle$ since:

$$\psi^{\#\dagger}\gamma^4\psi^{\#} = \left[\psi^\dagger\gamma^{4\#}\psi\right]^{\#} = \left[\psi^\dagger\left(-\gamma^4\right)\psi\right]^{\#} = -\psi^\dagger\gamma^4\psi \tag{21}$$

The spatial inversion (or mirroring operator M) which inverts all of the relative velocity vectors, is then (within an arbitrary phase factor):

$$M\psi\left(\mathbf{r}\right) = \gamma^4\psi^{\#}\left(-\mathbf{r}\right) \tag{22}$$

This operator inverts observables computed from γ^0, γ^5, and γ^4 independently of the change in sign of $\mathbf{r}$.

The Dirac equation for a particle in electromagnetic potentials is:

$$\left[\partial_t + c\gamma^5\sigma_i\partial_i + \tilde{\mathrm{i}}\Omega\gamma^0 + \mathrm{i}e\Phi - \mathrm{i}e\gamma^5\sigma_i A_i\right]\psi = 0 \tag{23}$$

The parity of the plain unit imaginary (i) is not yet determined. When applied to this equation, the parity operator inverts γ^0, γ^5, $\tilde{\mathrm{i}}$, and ∂_i (the matrices are inverted because they anti-commute with γ^4). Denoting spatially inverted quantities with subscript M, the spatially inverted Dirac equation is:

$$\left[\partial_t + c\gamma^5\sigma_i\partial_i + \tilde{\mathrm{i}}\Omega_M\gamma^0 + \mathrm{i}^{\#}e_M\Phi_M + \mathrm{i}^{\#}e_M\gamma^5\sigma_i A_{Mi}\right]\psi = 0 \tag{24}$$

We assume $\Omega_M = \Omega$ and $\mathrm{i}^{\#}e_M = \mathrm{i}e$. The transformed equation has the same form as the original Dirac equation except for the sign of the vector potential term.

$$\left[\partial_t + c\gamma^5\sigma_i\partial_i + \tilde{\mathrm{i}}\Omega\gamma^0 + \mathrm{i}e\Phi_M + \mathrm{i}e\gamma^5\sigma_i A_{Mi}\right]\psi_M = 0 \tag{25}$$

This sign change is necessary for consistency with gauge transformations. The gauge transformation

$$e\Phi' = e\Phi + \partial_t\chi \tag{26}$$

$$e\mathbf{A}' = e\mathbf{A} - \nabla\chi \tag{27}$$

$$\psi' = \psi\exp\left(-\mathrm{i}\chi\right) \tag{28}$$

suggests that the scalar potential may be regarded as a time derivative and the vector potential may be regarded as a spatial derivative. Taking $\Phi \equiv \partial_t g$ and $\mathbf{A} \equiv \nabla\times\mathbf{G} - \nabla g$ would leave the form of the equation invariant under spatial inversion:

$$\left[\partial_t + c\gamma^5\sigma_i\partial_i + \tilde{\mathrm{i}}\Omega\gamma^0 + \mathrm{i}e\partial_t g_M - \mathrm{i}e\gamma^5\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{G}_M - \nabla g_M\right)\right]\psi_M = 0 \tag{29}$$

The scalar and vector potentials must have opposite spatial inversion eigenvalues. We will assume that:

$$M\left[\mathrm{i}e\Phi\left(\mathbf{r}\right)\right] = -\mathrm{i}e\Phi\left(-\mathbf{r}\right) \tag{30}$$

$$M\left[\mathrm{i}e\mathbf{A}\left(\mathbf{r}\right)\right] = \mathrm{i}e\mathbf{A}\left(-\mathbf{r}\right) \tag{31}$$

The transformed Dirac equation is then:

$$\left[\partial_t + c\gamma^5\sigma_i\partial_i + \tilde{\mathrm{i}}\Omega\gamma^0 - \mathrm{i}e\Phi\left(-\mathbf{r}\right) + \mathrm{i}e\gamma^5\sigma_i A_i\left(-\mathbf{r}\right)\right]\psi_M = 0 \tag{32}$$

With these transformation properties, we will show that the new parity operator is consistent with an exchange of matter and anti-matter.

4. Eigenfunctions and eigenvalues

Next we consider the effect of the new parity operator on the eigenvalue equation. For simplicity we assume the vector potential $\mathbf{A}$ to be zero. Assuming temporal dependence $\exp(-\mathrm{i}Et)$, the eigenvalue equation is:

$$\left[-\mathrm{i}E + \mathrm{i}e\Phi + c\gamma^5 \boldsymbol{\sigma}\cdot\nabla\right]\psi = -\tilde{\mathrm{i}}\Omega\gamma^0\psi \tag{33}$$

The operator $\boldsymbol{\sigma}\cdot\nabla$ can be factored:

$$\boldsymbol{\sigma}\cdot\nabla\psi = \sigma_r\left[\partial_r + \mathrm{i}\frac{\boldsymbol{\sigma}}{r}\cdot(\mathbf{r}\times\nabla)\right]\psi = \sigma_r\left[\partial_r - \frac{\boldsymbol{\sigma}\cdot\mathbf{L}}{r}\right]\psi \tag{34}$$

The two-component angular solutions of the eigenvalue equations $\boldsymbol{\sigma}\cdot\mathbf{L}\Phi^{(+)}_{l,m}(\theta,\phi) = l\Phi^{(+)}_{l,m}(\theta,\phi)$ and $\boldsymbol{\sigma}\cdot\mathbf{L}\Phi^{(-)}_{l,m}(\theta,\phi) = -[l+2]\,\Phi^{(-)}_{l,m}(\theta,\phi)$ are well known [6] (l is the quantum number for orbital angular momentum). These two angular solutions are related by $\sigma_r\Phi^{(+)}_{l,m} = \Phi^{(-)}_{l,m}$ and yield opposite eigenvalues under coordinate inversion ($\mathbf{r}\to-\mathbf{r}$). Only the true scalar imaginary can appear within these functions.

Denote two wave functions as:

$$\psi^{(+)} = \frac{1}{r}\begin{bmatrix}\tilde{\mathrm{i}}G\Phi^{(+)}_{l,m}\\ F\Phi^{(-)}_{l,m}\end{bmatrix} \quad \text{and} \quad \psi^{(-)} = \frac{1}{r}\begin{bmatrix}\tilde{\mathrm{i}}F\Phi^{(-)}_{l,m}\\ G\Phi^{(+)}_{l,m}\end{bmatrix} \tag{35}$$

Each of these is an eigenfunction of the conventional parity operator, but they are exchanged by the new spatial inversion operator:

$$M\psi^{(+)} = (-)^l\,\psi^{(-)} \tag{36}$$

$$M\psi^{(-)} = (-)^{l+1}\,\psi^{(+)} \tag{37}$$

Using $\psi^{(+)}$ in the (original) Dirac equation with $\kappa \equiv l+1$ yields the coupled radial equations:

$$[E - e\Phi - \Omega]\,G + c\left[\partial_r + \frac{\kappa}{r}\right]F = 0 \tag{38}$$

$$[E - e\Phi + \Omega]\,F - c\left[\partial_r - \frac{\kappa}{r}\right]G = 0 \tag{39}$$

$\psi^{(-)}$ yields similar coupled equations with opposite sign of E and $e\Phi$, as expected for exchange of matter and anti-matter in a fixed external potential (to keep E positive we can assume the time dependence $\exp\left(+\tilde{\mathrm{i}}Et\right)$). The energy eigenvalues for $\psi^{(+)}$ in a negative Coulomb potential are therefore equal and opposite to the energy eigenvalues of $\psi^{(-)}$ in a positive Coulomb potential. The need for this result was the reason for assuming that the parity operator locally inverts the scalar potential term $\mathrm{i}e\Phi$.

5. Weak interactions

The projection operator for left-handed spinor components is:

$$\psi_L = \left(I - \gamma^5\right)\psi \tag{40}$$

The unit matrix I is a scalar and γ^5 is a pseudoscalar. However, the projection operator does not violate mirror symmetry so long as the reflected counterpart $\psi_R = \left(I + \gamma^5\right) M\psi$ is as physically plausible as the original projected wave function.

Weak interactions have a vertex factor of the form:

$$-\mathrm{i}\frac{g_w}{2\sqrt{2}}\gamma^\mu\left(I - \gamma^5\right) \tag{41}$$

It is sometimes said that this factor yields violation of mirror symmetry because it combines scalar and pseudoscalar components. However, mirror symmetry is violated if and only if the left-handed spinor does not have a mirror image counterpart in nature. Since the new spatial inversion operator exchanges matter and antimatter, all of the elementary particles involved in the weak interaction do in fact have spatially reflected counterparts in nature (electrons and positrons, left-handed neutrinos and right-handed anti-neutrinos, etc.). The mathematical form of the weak vertex factor is entirely consistent with mirror symmetry.

6. Comparison with conventional PC

The conventional PC operator is:

$$PC\psi\left(\mathbf{r}\right) = \mathrm{i}\gamma^0\gamma^2\psi^*\left(-\mathbf{r}\right) = \mathrm{i}\gamma^5\sigma_2\psi^*\left(-\mathbf{r}\right) \tag{42}$$

This differs from our spatial inversion operator M by an arbitrary phase factor, the factor of $\gamma^0\sigma_2$ and conjugation of the scalar imaginary (denoted by $\psi \to \psi^{*\#}$). The operator $\sigma_2\psi^{*\#}$ inverts spin, velocity ($\gamma^5\boldsymbol{\sigma}$), and the vectors computed from $\gamma^0\boldsymbol{\sigma}$ and $\gamma^4\boldsymbol{\sigma}$. The additional factor of γ^0 inverts velocity $\gamma^5\boldsymbol{\sigma}$ and the vector $\gamma^4\sigma$. Hence the net difference between PC and M is inversion of spin and the vector $\left\langle\gamma^0\boldsymbol{\sigma}\right\rangle$.

7. Time reversal

Physically, time reversal must invert the time derivative operator, velocity, and spin independently of the change in argument. One of the electromagnetic potentials must also be inverted. Velocity and spin are both inverted by the transformation (B for backward):

$$B\psi\left(t\right) \equiv \psi_B\left(t\right) = \sigma_2\psi^{*\#}\left(-t\right) \tag{43}$$

The velocity-representation space $(\gamma^5, \gamma^4, \gamma^0)$ is unaffected by this transformation. By contrast, the conventional time reversal operator $T\psi\left(t\right) = \mathrm{i}\sigma_2\psi^*\left(-t\right)$

inverts quantities associated with γ^4 but not other matrices of velocity-representation space. This suggests that the conventional time reversal operator is also incorrect. However, unlike the conventional parity transformation, there is no empirical evidence to validate this claim.

Applied to the Dirac equation, the new time reversal operator yields:

$$B\left\{\left[\partial_t + c\gamma^5\sigma_i\partial_i + \tilde{\mathrm{i}}\gamma^0\Omega + \mathrm{i}e\Phi - \mathrm{i}e\gamma^5\sigma_i A_i\right]\psi\right\} \tag{44}$$

$$= -\left[\partial_t + c\gamma^5\sigma_i\partial_i - \tilde{\mathrm{i}}\gamma^0\Omega - \mathrm{i}^{*\#}e_B\Phi_B - \mathrm{i}^{*\#}e_B\gamma^5\sigma_i A_{Bi}\right]\psi_B \quad = 0 \tag{45}$$

We recover the original form of the Dirac equation if $\Omega_B = -\Omega$ (i.e. Ω is an eigenvalue of an operator which transforms like a time derivative) and the potentials are interpreted as derivatives as discussed above for spatial inversion.

We assume the potentials to transform as:

$$B\left\{\mathrm{i}e\Phi\left(t\right)\right\} = \mathrm{i}e\Phi\left(-t\right) \tag{46}$$

$$B\left\{\mathrm{i}eA_i\left(t\right)\right\} = -\mathrm{i}eA_i\left(-t\right) \tag{47}$$

According to our interpretation of matter and anti-matter as mirror-images, time reversal does not exchange the two.

8. Combined Transformations

The combined MB transformation is:

$$MB\psi\left(\mathbf{r},t\right) = \gamma^4\sigma_2\psi^*\left(-\mathbf{r},-t\right) = \mathrm{i}\gamma^2\psi^*\left(-\mathbf{r},-t\right) \tag{48}$$

This is closely related to the conventional charge conjugation transformation C:

$$C\psi\left(\mathbf{r},t\right) = \gamma^4\sigma_2\psi^*\left(\mathbf{r},t\right) = \mathrm{i}\gamma^2\psi^*\left(\mathbf{r},t\right) \tag{49}$$

These differ only by inversion of the space and time arguments. In terms of dynamical behavior, charge conjugation has the same effect as inverting the sign of the electromagnetic potentials in the Dirac equation.

The conventional PT transformation is:

$$PT\psi\left(\mathbf{r},t\right) = \gamma^0\sigma_2\psi^*\left(-\mathbf{r},-t\right) \tag{50}$$

This differs from the new MB transformation by the gauge transformation factor γ^5, which rotates the velocity-representation space by 180 degrees.

The conventional PCT transformation is:

$$PCT\psi\left(\mathbf{r},t\right) = \gamma^5\psi\left(-\mathbf{r},-t\right) \tag{51}$$

This transformation is the conventional theoretical relation between matter and antimatter. Compared with the MB operator, it differs only by charge conjugation (which has similar effect to restoring the potentials inverted by MB) and by the factor γ^5 which inverts two vectors orthogonal to velocity.

9. Discussion

This explanation of the mirror symmetry of matter and antimatter has fundamental implications for the physical interpretation of matter. The most direct prediction of the new spatial inversion operator is the null prediction for supposed mirror processes associated with the conventional P operator. There is no reason to think that beta decay of matter should occur in both left- and right-handed versions. There is no reason to expect right-handed neutrinos or left-handed anti-neutrinos. For any oriented process in nature, mirror symmetry does not imply the existence of any mirror process other than the one which involves exchange of matter and antimatter.

Mirror symmetry implies that matter and antimatter differ only in their spatial structure. Since electrons and positrons are evidently mirror images, it is clear that these are not "point particles" but must have finite spatial structure. This structure is likely related to the oppositely signed electromagnetic potentials associated with positive and negative charges. If so, then one may speculate that since different particles can have the same long-range electromagnetic potentials, such particles may differ only in their short-range spatial structure near the center of each particle. Charge neutrality may be interpreted as a balance between left- and right-handed phenomena.

There have been proposals that "mirror matter", which is related to matter by the conventional P operator, may exist after all [7]. Some evidence seems to be consistent with this theory [8], but recent neutron experiments have yielded null results [9, 10]. While it is impossible to completely rule out the existence of a particle with particular physical properties, the existence of such particles is not a correct prediction of the hypothesis of mirror symmetry.

10. Conclusions

This derivation of the Dirac spatial inversion and time reversal operators provides a simple theoretical basis for the universally observed mirror symmetry between matter and antimatter. Unlike the conventional parity operator P, the spatial inversion operator M yields plausible physical processes consistent with mirror symmetry. This new operator explains the absence in nature of mirror-like phenomena which do not exchange matter and antimatter, without resorting to the invention of new unseen particles. The conventional parity operator has a flawed theoretical justification and requires the novel introduction of a mixed-parity vector space. In contrast, the new spatial inversion operator M and time reversal operator B are consistent with experimental evidence, ordinary geometry, and mirror symmetry.

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