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Chapter 4. Wave Refraction and Gravity

"It is only the relation of the magnitude to the instrument that we measure, and if this relation is altered, we have no means of knowing whether it is the magnitude or the instrument that has changed."
— Henri Poincaré, *Science et Méthode* 1897

4.1. Introduction

"*Gravity is probably due to a change in the structure of the aether, produced by the presence of matter."*
— George Francis FitzGerald 1894

Isaac Newton [Figure 4.1] published his theory of gravity in *Principia* in 1687. Newton realized that a force proportional to the inverse square of the distance between two masses would yield elliptical planetary orbits with the sun at one focus of the ellipse. He conjectured that the gravitational force might represent a tendency of matter to move from denser to rarer regions of the aether. Tests of Newton's theory were sometimes difficult and required planetary observational data accumulated over long periods of time. For example, in 1784 Pierre-Simon Laplace [Figure 4.2] determined that the apparently secular (non-periodic) motions of Jupiter and Saturn were actually periodic with a period of 929 years, the frequency corresponding to the difference between five periods of Saturn and two periods of Jupiter. Although Newton's law eventually succeeded in explaining most astronomical observations, a few observations resisted interpretation. This included the rate of rotation of the elliptical axes of Mercury.

Lóránd (or Roland) Eötvös [1891] [Figure 4.3] reported experimental results indicating that inertial mass and gravitational mass are exactly equal. Albert Einstein [1907] [Figure 4.4] then proposed the Principle of Equivalence between an accelerating reference frame and a gravitational field. He also deduced that the speed of light must vary in a gravitational field [Einstein 1911, 1912].

Harry Bateman [1909] observed that the condition for propagation of light:

$$c^2 dt^2 - dx^2 - dy^2 - dz^2 = 0 \qquad (1)$$

does not hold in a gravitational field. Instead a condition of the form:

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu = 0 \qquad (2)$$

describes the propagation of light in a gravitational field which is characterized by the coefficients $g_{\mu\nu}$. Time is denoted by x_0 and the coefficient g_{00} is equal and opposite to the spatial coefficients g_{ii} in the absence of gravity.

Albert Einstein and Marcel Grossmann [1913] proposed that particle motion in a gravitational field is described as a geodesic in space-time determined by the variational equation:

$$\delta \int ds = 0 \tag{3}$$

with *ds* defined as above. Combined with an equation relating the metric coefficients with the energy tensor of matter, this idea formed the General Theory of Relativity [Einstein 1915a,b,c]. David Hilbert [1915] [Figure 4.5] showed that the entire theory could be formulated using a variational principle. Karl Schwarzschild [1916] found exact solutions for a point mass.

Many predictions of the General Theory have been successfully validated by experimental observations. In addition to the usual attraction between massive objects, the theory also accurately predicts deflection of light rays around massive objects, deviations from simple elliptical planetary orbits, and non-Euclidean curvature of space. The theory also predicts the existence of black holes: regions where gravity is so strong that light cannot escape. There is now very strong astronomical evidence of black holes, including one at the center of our own galaxy.

In this chapter we will compare wave refraction with General Relativity. In particular, we will use the analogy of compression of a wave-carrying medium, such as an elastic solid. Other investigators have attempted to model the vacuum as an elastic solid. Two recent efforts are those of Hatch [1992] and Karlsen [1998]. Gravity has been interpreted by many as refraction due to a variable index of refraction of space [Alsing et al. 2001, Anonymous 2002, Colsman 1997, Evans et al. 2001, de Felice 1971, Peters 1974]. Although many physicists believe that gravity should have a quantum mechanical description, the classical description adequately explains a wide range of gravitational phenomena.

4.2. Wave propagation in a non-uniform medium

> "*It is worth noting that, strictly speaking, there cannot be any point particles in general relativity. They have to be much larger than their Schwarzchild radius ...*"
> — Hagen Kleinert [1989]

Since elastic waves yield bispinor equations similar to the equations of quantum mechanics, it is natural to question whether elastic waves can produce gravity. A simple mechanism is that twisting of the medium can generate tension which causes the medium to compress. This effect can be easily observed using a rubber band. Twisting the rubber band stretches it, thereby generating tension which pulls inward from the ends. The square of the wave speed is inversely proportional to density and therefore decreases as one approaches the region of increased density. Since waves refract in the direction of decreased wave speed there is a mutual attraction between rotational waves (see Figure []).

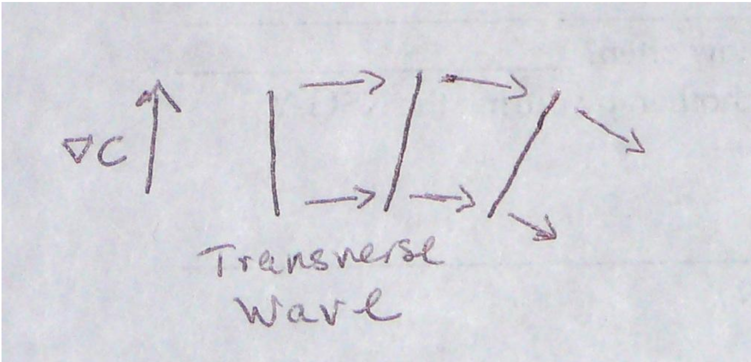

Figure: Waves refract toward the direction of slower wave speed. The rays are perpendicular to surfaces of constant phase.

4.2.1. Dispersion Relation and Metric Factors

Now consider the propagation of elastic waves in an ideal elastic medium with non-uniform density. For soliton waves the dispersion relation can be written as:

$$\omega^2 = c^2 k^2 + M^2 \tag{4}$$

The dispersion relation relates the various sources of phase shifts in the wave (time derivatives and spatial derivatives). The mass term represents the contribution of convection and rotation to the frequency, whereas *ck* represents the contribution of restoring forces or torques in the medium, resulting in wave propagation at the reduced frequency $\omega' = ck = \left(\omega^2 - M^2\right)^{1/2}$. In terms of the reduced frequency the dispersion relation appears to represent ordinary wave propagation:

$$\omega'^2 = c^2 k^2 \tag{5}$$

The condition of constant phase is:

$$\omega' dt - \mathbf{k} \cdot d\mathbf{l} = 0 \tag{6}$$

This equation describes wave propagation at speed c (since $dl/dt = \omega'/k = c$) with $d\mathbf{l}$ parallel to $\mathbf{k}$, as distinct from convection and rotation. In other words, a disturbance evolves over time due to convection and rotation of the medium (resulting in mass) and wave propagation (momentum). The wave propagation occurs with the characteristic wave speed as described above, but convection and rotation increase the frequency, thereby raising the phase velocity and reducing the group velocity.

It is customary to assume positive frequency, in which case the relative sign of the wave vector may need to be altered:

$$\omega' dt = \pm \mathbf{k} \cdot d\mathbf{l} \tag{7}$$

The phase velocity ($v_\Phi \ge c$) is:

$$v_\Phi = \frac{dl}{dt} = \frac{\omega}{k_l} = \frac{\omega}{\left(\omega^2 - M^2\right)^{1/2}} c \tag{8}$$

The group velocity ($v_G \le c$) is:

$$v_G = \frac{d\omega}{dk} = \frac{k}{\omega} c^2 = \frac{\left(\omega^2 - M^2\right)^{1/2}}{\omega} c \tag{9}$$

Using the propagation condition, we can define a 'phase separation' $d\chi$ for arbitrary space-time paths which measures the deviation from the propagation condition ($\omega' dt = \pm k dl$):

$$d\chi^2 = \omega'^2 dt^2 - k^2 dl^2$$

This differential separation should be zero for the true propagation path. The integrated phase separation is:

$$\int d\chi = \int \left[\omega'^2 dt^2 - k^2 dl^2\right]^{\frac{1}{2}} \tag{10}$$

If **k** and ω' are variable, then neighboring space-time paths must still yield equal phase shifts in order to maintain the transverse orientation of the wave. This condition yields the equation of a geodesic:

$$\delta \int d\chi = \delta \int \left[\omega'^2 dt^2 - k^2 dl^2\right]^{\frac{1}{2}} = 0 \tag{11}$$

Take reference values of ω_0 and k_0 with $\omega_0 / k_0 = c_0$. Multiplication of the geodesic equation by $1/k_0$ to obtain $ds = d\chi / k_0$ yields the geodesic equation in terms of the ordinary 'separation' ds:

$$\delta \int ds = \delta \int \left[\frac{\omega'^2}{\omega_0{}^2} c_0^2 dt^2 - \frac{k^2}{k_0^2} dl^2 \right]^{\frac{1}{2}} = 0 \tag{12}$$

This expression is equivalent to Einstein's formulation of general relativity [Einstein 1956 p. 78] if we assume a diagonal metric tensor with:

$$g^{tt} = \frac{\omega'^2}{\omega_0^2}$$
$$g^{xx} = g^{yy} = g^{zz} = -\frac{k^2}{k_0^2} \tag{13}$$

So that the geodesic equation is:

$$\delta \int \left[\sum_{\mu\nu} g^{\mu\nu} dx_\mu dx_\nu \right]^{1/2} = \delta \int \left[\sum_{\mu} g^{\mu\mu} dx_\mu^2 \right]^{1/2} = 0 \tag{14}$$

Hence Einstein's metric factors can be interpreted quite simply as the normalized values of the squared wave number and (reduced) frequency. Einstein's formulation is a bit more general in that it allows for non-isotropic metrics, and the above formula can be easily generalized to allow for independent variations of k_x/k_{0x}, k_y/k_{0y}, and k_z/k_{0z} (with appropriate dispersion relation). It is not clear that this generalization is important in nature, so it is not pursued here.

For simplicity, we rewrite the geodesic equation as:

$$\delta\int\left[\sum_{\mu} g^{\mu\mu}\left[\frac{dx_\mu}{d\tau}\right]^2\right]^{1/2} d\tau \equiv \delta\int f^{1/2} d\tau = 0 \tag{15}$$

The Euler-Lagrange equations are:

$$\frac{\partial}{\partial\tau}\left[f^{-1/2} g^{\nu\nu}\frac{dx_\nu}{d\tau}\right] - \frac{1}{2} f^{-1/2}\sum_{\mu}\frac{\partial g^{\mu\mu}}{\partial x_\nu}\left[\frac{dx_\mu}{d\tau}\right]^2 = 0 \tag{16}$$

Since the metric factors do not depend explicitly on the parameter τ, and $g_{\nu\nu} g^{\nu\nu} = 1$:

$$\frac{d^2 x_\nu}{d\tau^2} = \frac{1}{2} g_{\nu\nu}\frac{\partial g^{\mu\mu}}{\partial x_\nu}\left[\frac{dx_\mu}{d\tau}\right]^2 \tag{17}$$

For velocities small compared with the speed of light $\tau \approx c_0 t$ and $dt/d\tau \approx 1$:

$$\frac{d^2 x_i}{dt^2} = -\frac{c_0^2}{2}\frac{\partial g^{tt}}{\partial x_i} \tag{18}$$

The right hand side may be interpreted as gravitational acceleration and is equivalent to Einstein’s expression [Einstein 1956 p.89] except for a different sign convention (Einstein uses imaginary time, thereby changing the sign of the temporal metric component).

4.2.2. Relation between metric components

Consider the variations of ω and *k* for a refracting wave. For simplicity, consider the one dimensional problem of an elastic tube of radius *R* with varying density along its length. This tube could be an infinitesimally small region of a larger elastic solid. Consider a torsion wave with 1-D spatial profile of rotation angle Θ_z vs. *z*. Consider such a wave element with range of angles $d\Theta_z$ monotonically increasing over a distance *dz*.

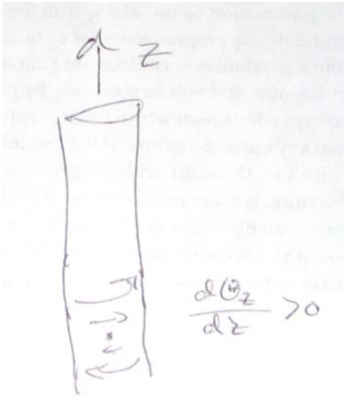

Figure: Local torsion wave propagating along z-axis.

For a one-dimensional inertial moment density $I_z = \rho R^2/2$ and elasticity coefficient $K_z = \mu R^2/2$ the angular momentum dL and energy $d\varepsilon$ of a local region are:

$$dL = I_z \frac{\partial \Theta_z}{\partial t} dz = I_z \frac{\partial \Theta_z}{\partial z}\frac{dz}{dt} dz = I_z c d\Theta_z$$
$$d\varepsilon = \left[\frac{1}{2} I_z \left[\frac{\partial \Theta_z}{\partial t} \right]^2 + \frac{1}{2} K \left[\frac{\partial \Theta_z}{\partial z} \right]^2 \right] dz \tag{19}$$

Each of these quantities must be conserved (convective time derivative equal to zero) as the wave propagates through the spatially-varying medium. Writing the expression for energy in terms of the constant angular momentum yields:

$$d\varepsilon = \frac{1}{2} dL \frac{\partial \Theta_z}{\partial t} + \frac{1}{2} K \left[\frac{\partial \Theta_z}{\partial z} \right]^2 dz = \frac{1}{2} dL \frac{\partial \Theta_z}{\partial t} + \frac{1}{2} K \left[\frac{\partial \Theta_z}{\partial z} \right] d\Theta_z \tag{20}$$

Energy conservation therefore requires that changes in wave characteristics be related by:

$$dL \Delta \left[\frac{\partial \Theta_z}{\partial t} \right] = -K d\Theta_z \Delta \left[\frac{\partial \Theta_z}{\partial z} \right] \tag{21}$$

Substituting the expressions for angular momentum $dL = I_z c d\Theta_z$ and initial wave speed $K/I_z = c^2$ yields:

$$\Delta \left[\frac{\partial \Theta_z}{\partial t} \right] = -c \Delta \left[\frac{\partial \Theta_z}{\partial z} \right] \tag{22}$$

In the Fourier domain the above expression becomes:

$$\Delta \omega = -c \Delta k \tag{23}$$

Since the local speed of light is $c = \omega/k$ (there is no mass in this one-dimensional case) we can write:

$$\frac{\Delta\omega}{\omega} = -\frac{\Delta k}{k} \tag{24}$$

For small changes we can use the approximation $\Delta\left[\omega^2/\omega_0^2\right] \approx 2\omega\,\Delta\omega/\omega_0^2 \approx 2\,\Delta\omega/\omega$ to get:

$$\Delta\left[\frac{\omega^2}{\omega_0^2}\right] = -\Delta\left[\frac{k^2}{k_0^2}\right] \tag{25}$$

Inclusion of mass merely changes ω to the reduced frequency ω'. In terms of metric components: $\Delta g_{tt} = \Delta g_{xx} = \Delta g_{yy} = \Delta g_{zz}$

This is in agreement with Einstein's result [Einstein 1956 p.89] except for the temporal sign convention. It follows from the fact that the Einstein tensor has zero divergence.

The change in wave speed is:

$$\Delta c = \frac{\omega_0 + \Delta\omega}{k_0 + \Delta k} - \frac{\omega_0}{k_0} = \frac{\Delta\omega}{k_0} - c_0\frac{\Delta k}{k_0} = 2\frac{\Delta\omega}{k_0} \tag{26}$$

This leads to the result:

$$\Delta g_{tt} = \Delta g_{xx} = \Delta g_{yy} = \Delta g_{zz} = \frac{\Delta c}{c_0} \tag{27}$$

Using this result in the expression for gravitational acceleration yields:

$$\frac{d^2 x_i}{dt^2} = -\frac{c_0}{2}\frac{\partial c}{\partial x_i} = -\frac{1}{4}\frac{\partial c^2}{\partial x_i} \tag{28}$$

Hence the gravitational acceleration is directly proportional to the gradient of the speed of light. The gravitational potential is:

$$U \approx \frac{c_0}{2}\Delta c \approx \frac{1}{4}\Delta c^2 = \frac{c^2 - c_0^2}{4} \tag{29}$$

where Δc^2 is the difference in the square of the speed of light from its unperturbed value. This expression for the gravitational potential is consistent with General Relativity [Einstein 1956 p. 84-93]. One may always offset this potential by a constant to make the values positive.

4.3. The gravitational potential

> *"The most incomprehensible thing about the universe is that it is comprehensible."*
> —Albert Einstein

The next question is whether compression yields the correct form of the gravitational potential. The equation of compression waves in an elastic solid is:

$$\frac{\partial^2}{\partial t^2}\rho = c^2\nabla^2\rho \tag{30}$$

Assuming the density to be slowly varying allows the time derivatives to be neglected:

$$\nabla^2\rho = 0 \tag{31}$$

Many large massive objects are nearly spherical in shape, implying only a radial dependence:

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\rho}{\partial r} = 0 \tag{32}$$

which has the solution:

$$\rho(r) = \rho_0\left[1+\frac{\eta}{r}\right] \tag{33}$$

where ρ_0 and η are constants. The speed of transverse waves is given by:

$$c^2(r) = \frac{\mu}{\rho} = \frac{\mu}{\rho_0\left[1+\frac{\eta}{r}\right]} \tag{34}$$

where μ is the shear modulus. The fractional variation of c^2 is given by:

$$\frac{\delta c^2}{c_0^2} = \frac{c^2(r)-c_0^2}{c_0^2} = \frac{1}{[1+\eta/r]} - 1 \approx -\frac{\eta}{r} + O\left(\frac{\eta}{r}\right)^2 \tag{35}$$

Hence the change of wave speed differs from the (1/*r*) dependence of the classical gravitational potential by the addition of higher order terms. However, even near the edge of the sun the variation is only $\delta c^2 / c_0^2 \approx -10^{-6}$, so the second order difference is extremely small.

The change in the speed of light is evidently caused by the presence of mass (*M*) and falls off inversely proportional to distance (*r*) away from a spherically symmetrical distribution of mass (except for very small distances). The expression for the Newtonian gravitational potential is:

$$U(r) = \frac{GM}{r} \tag{36}$$

Where $G = 6.673\times10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$ is the gravitational constant. Notice that the gravitational potential has units of velocity squared.

4.4. Consequences of gravity

4.4.1. Newtonian gravity

Given the form of the gravitational potential and the expression for acceleration in terms of variations in the speed of light, we can express the gravitational acceleration of an object in terms of the potential:

$$\frac{d^2 x_i}{dt^2} = -\frac{1}{4}\frac{\partial c^2}{\partial x_i} = -\frac{\partial U}{\partial x_i} \tag{37}$$

The acceleration is simply equal to the gradient of the gravitational potential, as in Newtonian gravity.

4.4.2. Bending of light

For propagation of light waves, we can no longer neglect changes in position relative to changes in time. Take the velocity in the x_3 direction to be c and the gradient in the speed of light to be along x_1 as in Figure [].

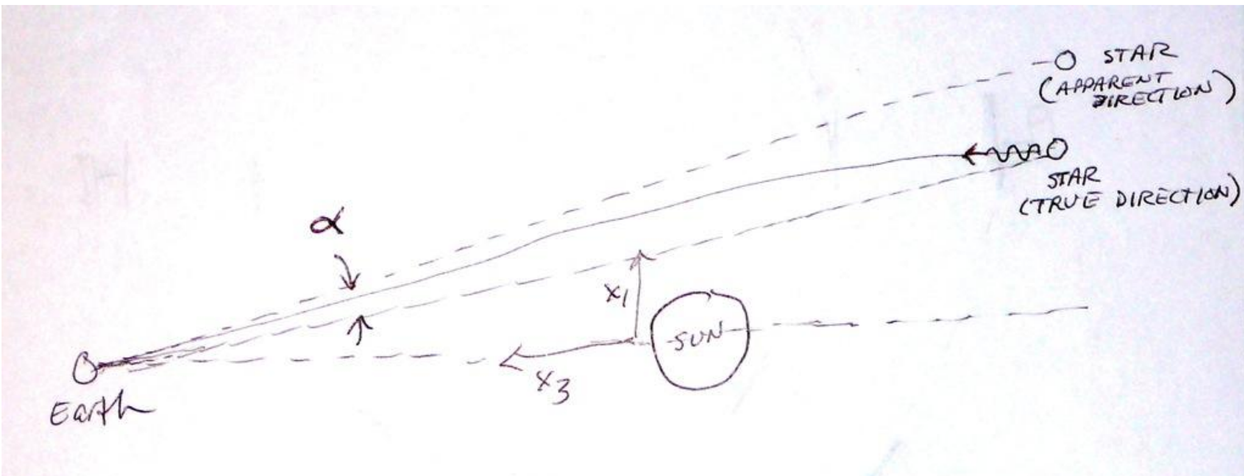

Figure: Apparent position of a star and the path of a light ray past the sun.

The acceleration is then:

$$\frac{d^2 x_1}{dt^2} = \frac{1}{2}c^2 g_{11}\frac{\partial g_{00}}{\partial x_1} + \frac{1}{2}c^2 g_{11}\frac{\partial g_{33}}{\partial x_1} = -c\frac{\partial c}{\partial x_1} = -2\frac{\partial U}{\partial x_1} \tag{38}$$

This is twice the Newtonian acceleration rate.
Integrating over a path with a $1/r$ gravitational potential yields:

$$\begin{aligned} \frac{dx_1}{dt} &= -2\int_{-\infty}^{\infty} dt\left[\frac{\partial U}{\partial x_1}\right] = \frac{2}{c}\int_{-\infty}^{\infty} dx_3\left[\mu\frac{\partial}{\partial x_1}\left(\frac{1}{x_1^2 + x_3^2}\right)^{1/2}\right] \\ &= -\frac{2}{c}\int_{-\infty}^{\infty} dx_3\left[\mu\left(\frac{x_1}{\left(x_1^2 + x_3^2\right)^{3/2}}\right)\right] = -\frac{2}{c}\mu\left[\frac{x_3}{x_1\sqrt{x_1^2 + x_3^2}}\right]_{x_3=-\infty}^{x_3=\infty} = -\frac{4\mu}{cx_1} \end{aligned} \tag{39}$$

The gravitational coefficient for the sun is:

$$\mu = \left(6.7 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}\right)\left(2.0 \times 10^{30} \text{kg}\right) = 1.4 \times 10^{20} \frac{\text{m}^3}{\text{s}^2} \tag{40}$$

This yields a perpendicular velocity of:

$$\frac{dx_1}{dt} = -\frac{4\mu}{cx_1} = \frac{4\left(1.4 \times 10^{20} \text{ m}^3/\text{s}^2\right)}{\left(3.0 \times 10^8 \text{ m }/\text{s}\right)x_1} = -\frac{1.9 \times 10^{12}}{x_1} \frac{\text{m}}{\text{s}} \tag{41}$$

Just outside the radius of the sun, x1=7.0x10^8 m:

$$\frac{dx_1}{dt} = -\frac{1.9 \times 10^{12}}{7.0 \times 10^8} \frac{\text{m}}{\text{s}} = -2.7 \times 10^3 \frac{\text{m}}{\text{s}} \tag{42}$$

The angle of deflection is:

$$\alpha = \frac{2.7 \times 10^3}{3.0 \times 10^8} = 9.0 \times 10^{-6} \text{radians} = 5.2 \times 10^{-4} \text{degrees} = 1.8'' \tag{43}$$

This deflection was first observed during a 1919 solar eclipse [Dyson, et al 1920]. More recent measurements use radio waves, which do not require waiting for eclipses [Lebach et al. 1995].

Since the light slows down near the sun, there is also a delay in the signal as compared with propagation in free space. This delay has also been measured and is in agreement with experiment [Shapiro et al. 1977, Bertotti et al. 2003].

4.4.3. Curvature of space

One supposedly bizarre prediction of general relativity is that “space is curved”. What this means is that measurements of geometrical shapes are not consistent with Euclidean geometry. For example, suppose we measure the circumference of a circle of radius R_1 by shining light past a series of mirrors orbiting in space as shown in Figure []. For simplicity, we will treat the earth as a point-like source of gravity.

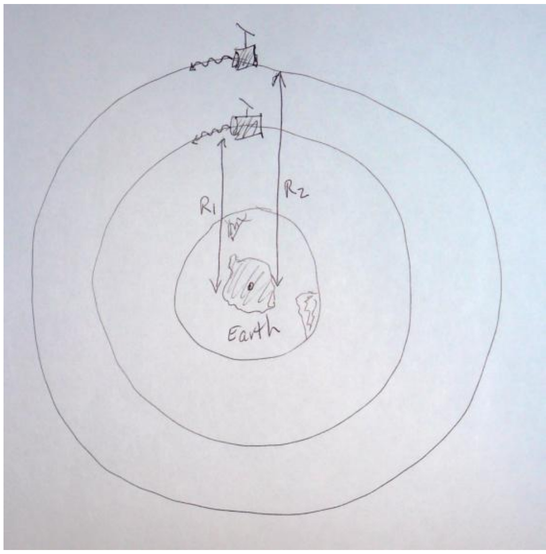

Figure: Distance measurements in a gravitational field.

We take the speed of light to be an approximation of the form derived above:

$$c(r) = c\frac{c_0}{[1+\eta/r]^{1/2}} \approx \frac{c_0}{1+\eta/2r}$$

Neglecting any delay during the reflection process, the light propagates with constant speed c_1 over a distance $2\pi R_1$, so that the propagation time is:

$$t_1 = \frac{2\pi R_1}{c_1} = \frac{2\pi}{c_0}\left[R_1 + \frac{\eta}{2}\right]$$

Since one cannot directly determine the absolute speed of light, the measured circumference L_1 is:

$$L_1 = c_0 t_1 = 2\pi\left[R_1 + \frac{\eta}{2}\right]$$

The circumference of a second circle with radius may be measured similarly. To avoid effects of different clock speeds, the transit time can be measured using the clock at R_1 by sending signals when the light wave is transmitted and when it is received by the satellite at R_2. The measured circumference of the circle at R_2 is:

$$L_2 = c_0 t_2 = 2\pi\left[R_2 + \frac{\eta}{2}\right]$$

The time of flight of light between the two circles is:

$$t_D = 2\int_{R_1}^{R_2} \frac{1}{c(r)} dr = \frac{2}{c_0}\int_{R_1}^{R_2}\left[1+\frac{\eta}{2r}\right]dr = \frac{2}{c_0}\left[R_2 - R_1 + \frac{\eta}{2}\ln\left[\frac{R_2}{R_1}\right]\right]$$

This means that the measured difference in radii is:

$$\Delta = \frac{c_0 t_D}{2} = R_2 - R_1 + \frac{\eta}{2}\ln\left[\frac{R_2}{R_1}\right]$$

According to Euclidean geometry, the two circumferences should be related by: $L_2 - L_1 = 2\pi\Delta$. Instead, we have:

$$\frac{2\pi\Delta}{L_2 - L_1} = 1 + \frac{\eta}{2}\frac{\ln[R_2/R_1]}{(R_2 - R_1)} > 1$$

Compared with Euclidean geometry, the measured circumference is smaller than expected for the measured change of diameter. This is the meaning of "curved space". However, the apparent curvature is actually attributable to the variation in the speed of light, which distorts the measurement of distances.

4.4.4. Black Holes

We saw above that light is deflected when it passes by a massive object such as the sun. If the gradient in the speed of light is large enough, then the light can become trapped. An object whose gravitational field is strong enough to trap light is called a "black hole".

For the geometry described above in Figure [] with variable x_1 replaced by *r* at the point of closest approach, the centripetal acceleration condition for trapping light is:

$$\frac{d^2r}{dt^2} = -\frac{c^2(r)}{r}$$

In terms of the gravitational potential, this condition is:

$$\frac{\partial U}{\partial r} = -\frac{4U(r)+c_0^2}{2r}$$

In terms of the mass of the black hole, for a this is:

$$\frac{GM}{r^2} = \frac{4GM/r - c_0^2}{2r} = 2\frac{GM}{r^2} - \frac{c_0^2}{2r}$$

Solving for r:

$$r = \frac{2GM}{c_0^2}$$

This radius is called the "Schwarzchild radius". Any light which reaches this point from the outside will be trapped.

Black holes were once considered an absurdity, but there is now a wealth of evidence for their existence in the universe.

4.5. Summary

"*The bigger they are the harder they fall.*"
— Anonymous

The above derivation demonstrates that gravity can be interpreted as wave refraction in a non-uniform medium. Unlike quantum theories in which gravity waves are assigned a spin of 2, the present model utilizes compression which is a scalar. There is absolutely no physical evidence indicating that gravity should be quantized.

Compression waves in a solid can in principle propagate at a speed equal or greater than the speed of transverse (or torsion) waves. Therefore it is quite possible that gravity waves propagate at a speed greater than *c*. If that is the case then the measured speed would also be direction dependent due to the earth's motion relative to the vacuum.

In summary, gravity may be interpreted as a description of wave refraction due to decreased velocity of light in the vicinity of matter. If the aether is taken to be an elastic solid, then the variation in light speed might be attributed to compression. The spatial metric components are interpreted as the ratio between the squared wave numbers at different positions. The temporal metric component is interpreted similarly as the ratio between squared frequencies at different positions. Conservation of angular momentum and energy yield the correct relation between spatial and temporal metric components. The derived form of the gravitational potential falls off as $1/r$ for large distances but also includes higher-order terms.

Gravitation deflects light in accordance with the laws of wave refraction. It also makes space appear to be non-Euclidean. Black holes bend light rays so strongly that the light becomes trapped. All of these effects are easily understood using the classical model of an elastic solid aether.

Exercises

1. Compute the expected delay of light propagating near the sun relative to propagation in free space.
2. Compute the Schwarzchild radius of an object with one solar mass.
3. By what factor would the solar density have to be increased to make the solar radius equal to the Schwarzchild radius?
4. For what mass would the Schwarzchild radius be 1Å? (1Å=10^{-10} m).
5. Compute the impact parameter (distance of closest approach for a straight line) for which light will be trapped in a black hole. This distance could be regarded as the measured, as opposed to absolute, Schwarzchild radius.

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